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Araştırma Makalesi / Research Article

Two Parameter Beta-Exponential Distribution: Properties and Applications in Demography and Geostandards

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Abstract

Modeling and analyzing lifespan data is essential in many application areas, including medicine, engineering, and finance. These types of data have been modeled using various lifetime distributions. The assumed probability model(s) have a significant impact on the efficiency of the procedures used in statistical research.Forthisreason, much work has been devotedtoderivingalargeclass of normal probability distributions andrelated statistical techniques. However,real-worlddatachallenge all established probability models, leaving manyimportant issues unresolved.This present work add another novel distribution with two parameter called two-parameter betaexponential distribution (TPBED), including the beta (2,b)distribution and the new XLindley distribution as special cases. We provide a complete mathematical treatment of this distribution. We derive the moment generating function and the r-th moment, thereby generalizing some results from the literature. Expressions for the density, moment generating function, entropy and the r-th moment of the order statisticare also obtained. We observe in three applications to simulated and real data sets**(**demography and g*eostandards)* that this model is quite flexible and can be used quite effectively for analyzing active data in place of one and two-parameter distributions such asthe exponential, Lindley, XLindley, new XLindley, Xgamma, Zeghdoudi, Chen, Lindley gamma, quasi-new Lindley, two-parameter Lindley, Power XLindley, and Gamma.

Keywords: Two parameter distribution, beta distribution, new Xlindley disribution, moments

1. Introduction

In many applied sciences such as medicine, engineering, and finance, among others, the modelling and analysis of life expectancy data is important. Several lifetime distributions have been used to model this type of data. The quality of the procedures used in statistical analysis depends heavily on the assumed model or probability distribution. For this reason, considerable efforts have been made to develop large classes of standard probability distributions as well as related statistical methods. However, there are still many important issues that do not hold up to classical or standard probability models. Some generalbeta distributions havebeenremoved.

Recently, some beta-generalized distributions have been considered. The beta-normal, beta-Frêchet, beta-Gumbel, beta-exponential, beta generalized half-normal, beta generalized Rayleigh, beta generalized exponential, and beta Lindley distributions, in that order, were put forth by (Eugene et al., 2002), (Nadarajah and Gupta, 2004),(Nadarajah and Kotz, 2004), (Nadarajah and Kotz, 2006),(Pescim et al., 2010),(Cordeiro et al., 2013), (Barreto-Souza et al., 2010)and (Merovci and Sharma, 2014). (Jones, 2004) explores this generic beta family and demonstrates that it has intriguing distributional characteristics as well as the possibility for fascinating statistical applications. Its order statistics serve as the motivation for this discussion.

In this paper, we introduce the two parameter beta-exponential distribution, a novel generalization of the new XLindley distribution. In the framework of Bayesian statistics, the new XLindley distribution was first put up by (Khodja et al., 2023).

(Khodja et al., 2023), they discussed the various statistical properties of new XLindley distribution. Furthermore, the research employs a Monte Carlo simulation to assess and compare the performance of various estimators in estimating the unknown parameter of the new XLindley distribution. This model was compared with many current distributions such as XLindley (Chouia and Zeghdoudi, 2021), Weibull, gamma, exponential, Zeghdoudi(Messaadia and Zeghdoudi, 2018), Akash (Rama, 2015), Lindley (Ghitany et al., 2008), Chris-Jerry (Onyekwere and Obulezi, 2022), Shanker, and Xgamma (Sen et al., 2016). Among all models, it is concluded that the new one-parameter distribution performed the best in modeling based on criteria such as the Akaike information criterion, Bayesian information criterion, and others. The cumulative distribution function (cdf) of the new XLindley distribution (NXLD) (Khodja et al., 2023) as follows:

$$F(x)=1-\left(\frac{\theta x}{2}+1\right)e^{-\theta x} \quad \text{where } x>0 \text{ and } \theta>0.$$

And the corresponding (pdf) defined as follows:

$$f(x)=\frac{\theta\left(1+\theta x\right)}{2}e^{-\theta x} \quad \text{where } x>0 \text{ and } \theta>0.$$

Here is how the rest of the paper is organized. In Section 2, the formulation of the proposed distribution is presented. Some distributional properties of the new model are discussed in Section 3. We give two real data sets to demonstrate the applicability of the proposed distribution in section 4. A simulation algorithm is provided in Section 5 to generate the random sample from two-parameter beta exponential distribution (TPBED).

1.1. Formulation of the two-parameter beta exponential distribution (TPBED)

Let F (x) denote the cumulative distribution function of a random variable X of the new XLindley distribution, and then the cumulative distribution function for a new class of distribution for the random variable X; as defined by (Eugene et al., 2002)is generated by applying the inverse (cdf) to a beta(2,b) distributed random variable to obtain

$$G(x)=\Gamma(2+b)\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^{2}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^{k}}{k!(2+k)\Gamma(b-k)}$$

And the (pdf) of TPBED

$$g(x)=\theta b(x\theta+1)(1+b)\sum_{j=0}^{1}(-1)^{1-j}\left(\frac{1}{2}\right)^{b+1-j}C_j^{\,1}(x\theta+2)^{b-j}e^{-\theta x(b+1-j)}$$

$$g(x)=\theta b(1+b)\left(x\theta+1\right)\sum_{j=0}^{1}(-1)^{1-j}\left(\frac{1}{2}\right)^{b+1-j}C_j^{\,1}(x\theta+2)^{b-j}e^{-\theta x(b+1-j)}$$

$$g(x)=\frac{\theta b(1+b)\left(x\theta+1\right)}{2}\sum_{j=0}^{1}(-1)^{1-j}C_j^{\,1}\left(\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^{b-j}e^{-\theta x}$$

We regard the series expansion as valid for $|z|<1$ and $\alpha>0$ real non integer

$$(1-z)^{\alpha-1}=\sum_{i=0}^{\infty}\frac{(-1)^{i}\Gamma(\alpha)}{i!\Gamma(\alpha-i)}z^{i}$$

We have

$$\left(1-\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)\right)^{b-j}=\sum_{i=0}^{\infty}\frac{(-1)^{i}\Gamma(b-j+1)}{i!\Gamma(b-j+1-i)}\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^{i}$$

Also, we have

$$\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^{i}=\sum_{s=0}^{i}\frac{(-1)^{s}\Gamma(i+1)}{s!\Gamma(i+1-s)}\left(\frac{x\theta}{2}+1\right)^{s}e^{-\theta sx}$$

By using the binomial expansion for $\left(\frac{x\theta}{2}+1\right)^{s}$

$$\left(\frac{x\theta}{2}+1\right)^{s}=\sum_{q=0}^{s}C_s^{\,q}\left(\frac{x\theta}{2}\right)^{q}$$

$$\left(1-\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)\right)^{b-j}=\sum_{i=0}^{\infty}\sum_{s=0}^{i}\sum_{q=0}^{s}C_s^{\,q}\frac{(-1)^{s+i}\Gamma(b-j+1)\Gamma(i+1)}{i!s!\Gamma(b-j+1-i)\Gamma(i+1-s)}\left(\frac{x\theta}{2}\right)^{q}e^{-\theta sx}$$

$$g(x)=b(1+b)\sum_{j=0}^{1}\sum_{i=0}^{\infty}\sum_{s=0}^{i}\sum_{q=0}^{s}C_s^{\,q}C_j^{\,1}\left(\frac{\theta}{2}\right)^{q+1}\frac{(-1)^{1+s+i-j}\,\Gamma(b-j+1)\Gamma(i+1)}{i!s!\Gamma(b-j+1-i)\Gamma(i+1-s)}(x\theta+1)x^q e^{-\theta x(s+1)}$$

We take

$$W=b(1+b)\sum_{j=0}^{1}\sum_{i=0}^{\infty}\sum_{s=0}^{i}\sum_{q=0}^{s}C_s^{\,q}C_j^{\,1}\left(\frac{\theta}{2}\right)^{q+1}\frac{(-1)^{1+s+i-j}\,\Gamma(b-j+1)\Gamma(i+1)}{i!s!\Gamma(b-j+1-i)\Gamma(i+1-s)}$$

Finally the (pdf) of TPBED is given by

$$g(x)=W(x^{q+1}\theta+x^q)e^{-\theta x(s+1)}$$

2.Statistical Properties

2.1.Moments

Proposition 1. If $X\to TPBED(b,\theta)$, the k th moment is given by:

$$E(X^k)=W\times\left(\frac{\theta\Gamma(k+q+2)}{(\theta(s+1))^{k+q+2}}+\frac{\Gamma(k+q+1)}{(\theta(s+1))^{k+q+1}}\right)$$

Proof. We have

$$E(X^k)=\int_0^\infty x^k g(x)dx$$

$$E(X^k)=W\times\int_0^\infty (x^{k+q+1}\theta+x^{k+q})e^{-\theta x(s+1)}dx$$

$$E(X^k)=W\times\left(\theta\int_0^\infty x^{k+q+1}e^{-\theta x(s+1)}dx+\int_0^\infty x^{k+q}e^{-\theta x(s+1)}dx\right)$$

By taking $v=\theta x(s+1)$ than $x=\dfrac{v}{\theta(s+1)}$

Finally, we have

$$E(X^k)=W\times\left(\frac{\theta\Gamma(k+q+2)}{(\theta(s+1))^{k+q+2}}+\frac{\Gamma(k+q+1)}{(\theta(s+1))^{k+q+1}}\right)$$

2.2. Moments generating function

The (mgf) of the two-parameter beta exponential distribution (TPBED) is given by

$$M(t)=E(e^{tX})=\int_0^\infty e^{tX}g(x)dx$$

$$M(t)=W\times\int_0^\infty (x^{q+1}\theta+x^q)e^{-(\theta(s+1)-t)x}dx=W\times\left(\theta\int_0^\infty x^{q+1}e^{-(\theta(s+1)-t)x}dx+\int_0^\infty x^q e^{-(\theta(s+1)-t)x}dx\right)$$

$$M(t) = W \times \left(\frac{\theta\Gamma(q+2)}{(\theta(s+1)-t)^{q+2}} + \frac{\Gamma(q+1)}{(\theta(s+1)-t)^{q+1}} \right)$$

2.3. Entropy

Most people agree that the degree of uncertainty in a probability distribution may be determined using information and entropy. Nonetheless, a lot of correlations have been developed using entropy's properties. The fluctuation in uncertainty is measured by the entropy of a random variable X. The definition of Rényi's entropy is as follows:

$$R(t) = \frac{1}{1-t} \log \left\{ \int_0^\infty g^t(x)dx \right\}$$

Where t is integer greater than 0 and $t \neq 1$, for two-parameter betaexponential distribution (TPBED), we have:

$$R(t) = \frac{1}{1-t} \log \left\{ \int_0^\infty W^t (x^{q+1}\theta + x^q)^t e^{-\theta t x(s+1)} dx \right\}$$

Using the binomial expansion for $(x^{q+1}\theta + x^q)^t$ we get

$$(x^{q+1}\theta + x^q)^t = \sum_{l=0}^{t} C_t^l x^{q(2t-l)+t} \theta^t$$

So

$$R(t) = \frac{1}{1-t} \log \left\{ W^t \sum_{l=0}^{t} C_t^l \theta^t \int_0^\infty x^{q(2t-l)+t} e^{-\theta t x(s+1)} dx \right\}$$

$$= \frac{1}{1-t} \log \left\{ \frac{W^t \sum_{l=0}^{t} C_t^l \theta^t}{(\theta t(s+1))^{q(2t-l)+t+1}} \int_0^\infty y^{q(2t-l)+t} e^{-y} dy \right\}$$

Finally the Rényi's entropy of two-parameter betaexponential distribution (TPBED) is given by:

$$R(t) = \frac{1}{1-t} \log \left\{ \frac{W^t \sum_{l=0}^{t} C_t^l \theta^t}{(\theta t(s+1))^{q(2t-l)+t+1}} \Gamma(q(2t-l)+t+1) \right\}$$

2.4. Incomplete moments

The k th incomplete moment of X can be expressed as follows, according to similar computations:

$$T_k(t) = E(X^k / X < t) = \frac{1}{G(t)} \int_0^t x^k g(x)dx$$

$$= \frac{W}{G(t)} \int_0^t (x^{k+q+1}\theta + x^{k+q}) e^{-\theta x(s+1)} dx = \frac{W}{G(t)} \left(\theta \int_0^t x^{k+q+1} e^{-\theta x(s+1)} dx + \int_0^t x^{k+q} e^{-\theta x(s+1)} dx \right)$$

$$= \frac{W}{G(t)} \left(\frac{\theta}{(\theta(s+1))^{k+q+2}} \int_0^{t\theta(s+1)} y^{k+q+1} e^{-y} dy + \frac{1}{(\theta(s+1))^{k+q+1}} \int_0^{t\theta(s+1)} y^{k+q} e^{-y} dy \right)$$

$$= \frac{W}{G(t)}\left(\frac{\theta}{\left(\theta(s+1)\right)^{k+q+2}} \gamma\left(k+q+2, t\theta(s+1)\right) + \frac{1}{\left(\theta(s+1)\right)^{k+q+1}} \gamma\left(k+q+1, t\theta(s+1)\right) \right)$$

Finally,

$$T_k(t) = \frac{W}{G(t)\left(\theta(s+1)\right)^{k+q+1}} \left(\frac{\gamma\left(k+q+2, t\theta(s+1)\right)}{s+1} + \gamma\left(k+q+1, t\theta(s+1)\right) \right)$$

2.5. Stress-strength reliability

One way to calculate the stress-strength reliability R for a component with independent strength and stress random variables X and Y, following the two-parameter beta exponential distribution (TPBED) with parameters θ_1 and θ_2, respectively, is as follows:

$$R = P\left(Y < X\right) = \int_0^\infty g\left(x, \theta_1\right) G\left(x, \theta_2\right) dx$$

2.6. Order statistics

The i th order statistic of a sample is its i th smallest value. For a sample of size n, the n th order statistic (or largest order statistic) is the maximum; that is

$$X_{(n)} = \left\{X_1, X_2, ..., X_n\right\}$$

The sample range is the difference between the maximum and minimum. It is clearly a function of the order statistics:

$$X_{(n)} - X_{(1)} = range\left\{X_1, X_2, ..., X_n\right\}$$

We know that if $X_1 \le X_2 \le ... \le X_n$, denotes the order statistic of a random sample $X_1, X_2, ..., X_n$ from a continuous population with (cdf) $G(x)$ and (pdf) $g(x)$, then the (pdf) of $X_{(i)}$ is given by

$$g_{X_{(i)}}(x) = \frac{n!}{(i-1)!(n-i)!} g(x)\left(G(x)\right)^{i-1}\left(1-G(x)\right)^{n-i}$$

The (pdf) of the i th order statistic for the two-parameter beta exponential distribution (TPBED) is given by

$$g_{X_{(i)}}(x) = \frac{n!W(x^{q+1}\theta + x^q)e^{-\theta x(s+1)}}{(i-1)!(n-i)!} \left(\Gamma(2+b)\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^2 \sum_{k=0}^{\infty} \frac{(-1)^k\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^k}{k!(2+k)\Gamma(b-k)} \right)^{i-1} \times$$

$$\left(1-\Gamma(2+b)\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^2 \sum_{k=0}^{\infty} \frac{(-1)^k\left(1-\left(\frac{x\theta}{2}+1\right)e^{-\theta x}\right)^k}{k!(2+k)\Gamma(b-k)} \right)^{n-i}$$

3. Illustrations with Simulated and Real Datasets

3.1. Simulated data

In this part, we offered an approach to produce a random sample for the specified sample size (n) and parameter values of the two-parameter beta exponential distribution (TPBED). The steps involved in the simulation process are as follows.

- Step1. Set $n=35$ and $\Theta(b=1,\theta=3)$.
- Step2. Set initial value $x^0<1$ and $k=0$.
- Step3. Generate U~Uniform$(0,1)$,it's mean $U=x$.
- Step4. Update x^0 by using Newton's formula such as $x^*=x^0-\left.\left(\frac{G_\Theta(x)-U}{g_\Theta(x)}\right)\right|_{x=x^0}$.
- Step5. if $\left|x^*-x^0\right|\le\varepsilon$, then we take this value x^*.
- Step6. if $\left|x^*-x^0\right|>\varepsilon$, then we change the initial value, and go to step 4.
- Step7. repeat steps 4 to 6, and obtained random sample from $G(x)$.

We created a sample from the two-parameter betaexponential distribution (TPBED) with size $n=35$ using the prior algorithm. The simulated sample is given by:

0.1, 0.28252, 0.42337, 0.54985, 0.67905, 0.83465, 1.0757, 1.6796, 0.2, 0.35618, 0.48759, 0.61321, 0.75163, 0.93779, 1.2881, 2.7059, 0.3, 0.43825, 0.56417, 0.69491, 0.85603, 1.1155, 1.8245, 0.4, 0.52776, 0.65513, 0.80347, 1.0210, 1.5061, 0.5, 0.62596, 0.76704, 0.96159, 1.3455, 3.124

The variance-covariance matrix $I^{-1}\left(\hat{b},\hat{\theta}\right)$ of the MLEs under the modified beta-exponential distribution for simulated data is computed as

$$\begin{pmatrix} 0.172078 & -1.128508 \\ -1.128508 & 7.651671 \end{pmatrix}$$

The variances of the MLE of b and θ under the TPBED for simulated data are $\mathrm{var}\left(\hat{b}\right)=0.172078$, and $\mathrm{var}\left(\hat{\theta}\right)=7.651671$. Thus, 95% the confidence intervals for b and θ are $[0,1.279243]$ and $[0,9.717297]$ respectively.

Table 1.The ML estimates,-2 log-likelihood, AIC, BIC,AICC, and HQIC for Simulated data.

Model	$\hat{b}$	$\hat{\theta}$	$\hat{\gamma}$	AIC	BIC	$-2L$	$AICC$	$HQIC$
exponential		1.129		63.496	65.051	61.496	63.617	64.033
Lindley		1.568		61.079	62.635	59.079	61.201	61.616
XLindley		1.332		62.934	64.489	60.934	63.056	63.471
New-XLindley		1.749		59.626	61.182	57.626	59.747	60.163
Xgamma		1.930		65.001	66.557	63.001	65.123	65.538
Zeghdoudi		2.735		52.254	53.809	50.254	52.375	52.791
Chen		0.814	0.545	64.945	68.056	60.945	65.320	66.019
gamma Lindley		29.326	2.241	54.582	57.693	50.582	54.957	55.656
new quasi Lindley		2.144	44.158	55.586	58.697	51.586	55.961	56.660
two parameter Lindley I		2.186	37.857	55.016	58.127	51.016	55.391	56.090
Power XLindley		2.849	0.688	117.67	120.78	113.67	118.04	118.74
Gamma		2.673	2.367	53.729	56.840	49.729	54.104	54.803
TPBED	0.466	4.296	/	53.262	56.373	49.262	53.637	54.336

3.2. Real data analysis

Two actual datasets are used in this section to illustrate the new two-parameter model's practicality. The first actual dataset I shows the population of the United States (in millions) as recorded by the decennial census for the period 1790--1970.(McNeil and Tukey, 1977)

3.93, 5.31, 7.24, 9.64, 12.90, 17.10, 23.20, 31.40, 39.80, 50.20, 62.90, 76.00, 92.00, 105.70, 122.80, 131.70, 151.30, 179.30, 203.20

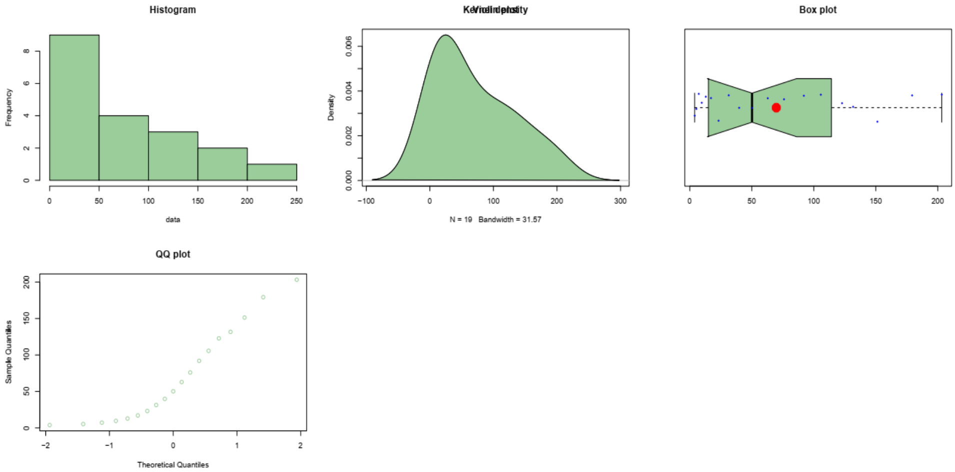

Figure 1. Histogram, kern density, box plot and QQ plot of the set data I.

And the second dataset II shows a numeric vector of 31 determinations of nickel content (ppm) in a Canadian syenite rock(Abbey, 1988).

5.2, 6.5, 6.9, 7.0, 7.0, 7.0, 7.4, 8.0, 8.0, 8.0, 8.0, 8.5, 9.0, 9.0, 10.0, 11.0, 11.0, 12.0, 12.0, 13.7, 14.0, 14.0, 14.0, 16.0, 17.0, 17.0, 18.0, 24.0, 28.0, 34.0, 125.0

Table 2. The ML estimates,-2 log-likelihood, AIC, BIC,AICC, and HQIC for dataset I.

Model	$\hat{b}$	$\hat{\theta}$	$\hat{\gamma}$	AIC	BIC	$-2L$	$AICC$	$HQIC$
exponential		0.014		201.32	202.26	199.32	201.55	201.48
Lindley		0.028		207.63	208.57	205.63	207.86	207.79
XLindley		0.028		206.92	207.87	204.92	207.16	207.08
New-XLindley		0.021		201.65	202.60	199.65	201.89	201.81
Xgamma		0.040		215.44	216.38	213.44	215.67	215.60
Zeghdoudi		0.043		220.98	221.92	218.98	221.22	221.14
Chen		0.297	0.027	203.96	205.85	199.96	204.71	204.29
gamma Lindley		0.017	0.024	203.23	205.12	199.23	203.98	203.55
new quasi Lindley		0.024	0.001	204.55	206.44	200.55	205.29	204.87
two parameter Lindley		0.022	44.26	203.70	205.59	199.70	204.45	204.02
Power XLindley		1.056	0.194	251.30	253.19	247.30	252.05	251.62
Gamma		0.014	1.006	203.32	205.21	199.32	204.07	203.64
TPBED	0.012	1.241		202.88	204.77	198.88	203.63	203.20

Table 3. The ML estimates,-2 log-likelihood, AIC, BIC,AICC, and HQIC for dataset II.

Model	$\hat{b}$	$\hat{\theta}$	$\hat{\gamma}$	AIC	BIC	$-2L$	$AICC$	$HQIC$
exponential		0.062		235.93	237.36	233.93	236.06	236.39
Lindley		0.118		231.49	232.92	229.49	231.62	231.95
XLindley		0.113		232.60	234.04	230.60	232.74	233.07
New-XLindley		0.095		235.48	236.92	233.48	235.62	235.95
Xgamma		0.172		237.91	239.34	235.91	238.05	238.38
Zeghdoudi		0.182		234.48	235.91	232.48	234.62	234.95
Chen		0.319	0.087	253.08	255.95	249.08	253.51	254.01
gamma Lindley		0.125	13.46	232.14	235.01	228.14	232.57	233.07
new quasi Lindley		0.125	7.013	232.05	234.92	228.05	232.48	232.99
two parameter Lindley I		0.112	0.389	233.10	235.96	229.10	233.52	234.03
Power XLindley		1.178	0.252	318.53	321.40	314.53	318.96	319.46
Gamma		0.117	1.867	231.93	234.80	227.93	232.36	232.87
TPBED	0.236	0.393		227.53	230.40	223.53	227.96	228.46

4.Conclusion

We have proposed TPBED as a generalizationof the new XLindley distribution discussed by(Khodja et al., 2023). We provide a mathematical treatment of this distribution thatincludes the density of the order statistics. We derive amomentgenerating function and provide aninfinitesummationof the moments of the new distribution and its order statistics. Three applications of TPBED are given to show that this distribution canprovidea better fit than other sub models discussed in the literature such asexponential, Lindley, XLindley, new XLindley, Xgamma, Zeghdoudi, Chen, Lindleygamma,quasi-new Lindley, two-parameter Lindley, Power XLindley, and Gamma. We hope that this generalization can attract broader applications in reliability, biology, and actuarial science.

Declaration of Author Contributions

The authors declare that they have contributed equally to the article. All authors declare that they have seen/read and approved the final version of the article ready for publication.

Declaration of Conflicts of Interest

All authors declare that there is no conflict of interest related to this article.

References

Abbey, S., 1988. Robust measures and the estimator limit. *Geostandards Newsletter*, 12(2): 241–248.

Barreto-Souza, W., Santos, A.H.S., Cordeiro, G.M., 2010. The beta generalized exponential distribution. *Journal of Statistical Computation and Simulation*, 80(2): 159–172.

Chouia, S., Zeghdoudi, H., 2021. The xlindley distribution: Properties and application. *Journal of Statistical Theory and Applications*, 20(2): 318–327.

Cordeiro, G.M., Cristino, C.T., Hashimoto, E.M., Ortega, E.M.M., 2013. The beta generalized Rayleigh distribution with applications to lifetime data. *Statistical Papers*, 54: 133–161.

Eugene, N., Lee, C., Famoye, F., 2002. Beta-normal distribution and its applications. *Communications in Statistics-Theory and Methods*, 31(4): 497–512.

Ghitany, M.E., Atieh, B., Nadarajah, S., 2008. Lindley distribution and its application.*Mathematics and Computers in Simulation*, 78(4): 493-506.

Jones, M.C., 2004. Families of distributions arising from distributions of order statistics. *Test*, 13: 1–43.

Khodja, N., Gemeay, A.M., Zeghdoudi, H., Karakaya, K., Alshangiti, A.M., Bakr, M.E., Balogun, O.S., Muse, A.H., Hussam, E., 2023. Modeling voltage real data set by a new version of Lindley distribution. *IEEE Access*, 11: 67220–67229.

McNeil, D., Tukey, J.W., 1977. Interactive data analysis : a practical primer. 186.

Merovci, F., Sharma, V.K. 2014. The Beta-Lindley Distribution: Properties and Applications. *Journal of Applied Mathematics*, 2014(1): 198951.

Messaadia, H., Zeghdoudi, H., 2018. Zeghdoudi distribution and its applications. *International Journal of Computing Science and Mathematics*, 9(1): 58–65.

Nadarajah, S., Gupta, A.K., 2004. The beta Fréchet distribution. *Far East Journal of Theoretical Statistics*, 14(1): 15–24.

Nadarajah, S., Kotz, S., 2004. The beta Gumbel distribution. *Mathematical Problems in Engineering*, (4): 323–332.

Nadarajah, S., Kotz, S., 2006. The beta exponential distribution. *Reliability Engineering and System Safety*, 91(6): 689–697.

Onyekwere, C.K., Obulezi, O.J., 2022. Chris-Jerry distribution and its applications. *Asian Journal of Probability and Statistics*, 20(1): 16–30.

Pescim, R.R., Demétrio, C.G. B., Cordeiro, G.M., Ortega, E.M.M., Urbano, M.R., 2010. The beta generalized half-normal distribution. *Computational Statistics and Data Analysis*, 54(4): 945–957.

Rama, S., 2015. Akash distribution and its applications. *International Journal of Probability and Statistics*, 4(3): 65–75.

Sen, S., Maiti, S.S., Chandra, N., 2016. The xgamma distribution: statistical properties and application. *Journal of Modern Applied Statistical Methods*, 15(1): 38.

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