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Araştırma Makalesi/ Research Article

Novel Two Parameter Model: Statistical Properties, Fuzzy Reliability and Applications

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Abstract

The improvement of one-parameter lifetime distributions together with Lindley, Zegdoudi, XLindley, new XLindley, XGamma, etc., has usually been popular, however including a brand new parameter to a distribution makes it better and more flexible than the present one. This paper presents a new family of two-parameter (NTPFD) derivatives of the two parameter polynomial exponential family. This new version is a generalization of numerous novel one-parameter distributions such as: the XLindley, new XLindley, and ZLindley distributions. We study the main statistical properties of NTPFD: the shape of the density and hazard rate functions, moments, skewness, kurtosis, coefficient of variation, Bonferroni and Lorenz curves, reliability parameters, stochastic ordering, entropies, and quantile function. A discussion of fuzzy reliability is also given. To study the applicability, usefulness, reliability and superiority of the proposed distribution over existing distributions, two real data sets are assessed and fitted to the NTPFD distribution and potential competitors such as: an uncensored data set corresponding to the remission duration of a random sample of 128 patients with bladder cancer and the US population recorded in the decennial census during the period 1790-1970.

Keywords: Two parameter distribution, moments, fuzzy reliability, application

1. Introduction

Exponential and Lindley distributions are two classic life expectancy distributions for modeling life expectancy data. Lindley (1958) introduced a life expectancy distribution called the Lindley distribution. Lindley distribution, which is a convex combination of exponential and gamma distributions, is a better fit and more flexible than exponential distribution but only one parameter does not allow to model all phenomena. For this, this work proposes a new family with two parameters. The distribution of X is a two-parameter family and the probability density function can be written as

$$f(t;\theta,\gamma) = b(\theta,\gamma)(a_0 + a_1 x))exp^{(-c(\theta,\gamma)t)}$$

where *b(θ,γ), a=a (θ,γ), a=a (θ,γ)* and *c(θ,γ)* are real-valued functions on [0,+∞[see(Belili et al., 2023) .We can check immediately:

- It is non-negative for *t>0*
- $P[a < x < b] = \int_a^b f(x;\theta,\gamma)\,dx$
- $\int_0^\infty f(x;\theta,\gamma)\,dx = 1$

In this paper, we proposed a special case of two-parameter family called two parameter Lindley family (NTPFD)

when $a_0 = \gamma, a_1 = (1-\gamma)\theta \ , c = \theta$

The density is $f(x;\theta,\gamma) = \theta(\gamma + (1-\gamma)\theta x)exp^{-\theta x}$

The corresponding cumulative distribution function (CDF), the survival function (SF) and hazard rate function (HRF) are given by

$$F(x;\theta,\gamma) = 1 - e^{-\theta x}(1 + (1-\gamma)\theta^2 x), x,\theta \succ 0.\ 0 \prec \gamma \prec 1$$

$$S(t;\theta,\gamma) = e^{-\theta x}(1 + (1-\gamma)\theta^2 x), x,\theta \succ 0.\ 0 \prec \gamma \prec 1$$

$$h(t;\theta,\gamma) = \frac{\theta^2(\gamma + (1-\gamma)\theta x)}{(1-\gamma)\theta^2 x + 1}$$

This distribution includes several distributions with one parameter such as:

- The XLindley distribution Chouia and Zeghdoudi (2021));
- The new XLindley distribution Khodja et al. (2023));
- The ZLindley distribution Saadia et al. (2024)).

The following is the format of this research paper:

Section 2 covers survival and hazard functions, moments, Stochastic Order, Entropies and other statistical properties. Sections 3 consider the Fuzzy reliability. Finally, two specific applications demonstrate the superior performance of the new family of models (NTPFD) over the two-parameter Lindley , Quasi Lindley, Power XLindley, TPQED distributions

2. A general theoretical result

2.1. Asymptotic behavior

This subsection discusses the shape characteristics of the PDF and HRF, respectively of the NTPFD . The behavior of NTPFD $at\ x = 0\ and\ x = \infty$, respectively, are given by

$$\lim_{x \to 0} f(x;\theta,\gamma) = \frac{\gamma\theta^2}{\theta\gamma + (1-\gamma)\theta} = \gamma\theta$$
$$\lim_{x \to \infty} f(x;\theta,\gamma) = 0.$$

The behavior of $h(x;\theta,\gamma)\ at x = 0\ and x = \infty$, respectively, are given by

$$\lim_{x \to 0} h(x;\theta,\gamma) = \frac{\gamma\theta^2}{\theta\gamma + (1-\gamma)\theta} = \gamma\theta$$

$$\lim_{x \to \infty} h(x;\theta,\gamma) = c(\theta,\gamma).$$

The following proposition states that there are two shapes for the PDF of the two-parameter polynomial exponential distribution, depending on the range of the parameters θ and γ.

Proposition 1. The PDF $f(x;\theta,\gamma)\ in\ (ref{:}PDF)$ of the NTPFD is

1.

2. Decreasing if $(1-\gamma)\theta - \theta\gamma > 0$.

3. Unimodel if $(1-\gamma)\theta - \theta\gamma > 0$.

Proof. The first and the second derivative of the PDF is determined as follows

$$\frac{df(x;\theta,\gamma)}{dx} = -\theta^2 e^{-\theta x}(2\gamma - 1 + (1-\gamma)\theta x)$$
$$\frac{d^2 f(x;\theta,\gamma)}{dx^2} = -\theta^3 e^{-\theta x}(3\theta - 2 + (1-\gamma)\theta x)$$

by equating last equation to zero and solve it with respect to x, we have:
the solution is $x = \frac{1-2\gamma}{(1-\gamma)\theta}$then, our critical point is $x^* = \frac{1-2\gamma}{(1-\gamma)\theta}, if\ (1-\gamma)\theta - \theta\gamma > 0$.

$$\frac{d^2 f(x;\theta,\gamma)}{dt^2} = \frac{\theta^2}{(1-\gamma)(\theta+1)} e^{-\theta t}((1-\gamma)\theta^2 t + \gamma(\theta - 2)), \frac{d^2 f(x;\theta,\gamma)}{dt^2} < 0,$$

then, $\forall\theta,\gamma > 0, x = \frac{(1-\gamma)\theta - \theta\gamma}{(1-\gamma)\theta^2}$ is the unique critical point which maximize the PDF (ref:PDF) and the PDF is unimodel.

Therefore, the mode of TPFD is defined as follows

$$M^* = \frac{1-2\gamma}{(1-\gamma)\theta}.$$

If $(1-\gamma)\theta - \theta\gamma < 0$, the PDF is decreasing, the mode will be

$$M^* = \frac{\gamma\theta^2}{\theta\gamma + (1-\gamma)\theta} = \gamma\theta.$$

Proposition 2. Let $h(x;\theta,\gamma)$ be the hazard rate function of of the TPFD. Then $h(x;\theta,\gamma)$ increasing.

Proof. The first derivative of $h(x;\theta,\gamma)$ is

$$\frac{dh(x;\theta,\gamma)}{dx} = \frac{\theta^2(1-\gamma)^2\theta^2}{((1-\gamma)\theta^2 x + \theta\gamma + (1-\gamma)\theta)^2} = \frac{(1-\gamma)^2\theta^4}{((1-\gamma)\theta^2 x + \theta)^2} = \frac{(1-\gamma)^2\theta^2}{(1-\gamma)\theta x + 1)^2}$$

It is easy to check that $h(x;\theta,\gamma)$ is an increasing function.

Moments and related measures of NTPFD

Let $X \sim NTPFD$, Then the i th moment of X is determined as follows

$$E(X^i) = \frac{\Gamma(1+i)}{\theta^{2+i}}[\theta\gamma + (1-\gamma)\theta(1+i)]$$

Hence, the first four moments of the $NTPFD$ random variable can be found by substituting i=1,2,3,4, respectively, in above equation. They are used to determine variance, Skewness, Kurtosis and coefficient of variation of $NTPFD$, respectively, as follows

$$Var(X) \ = \ E(X^2) - E(X) = \frac{2\gamma\theta^3 + 6(1-\gamma)\theta^3 - \gamma^2\theta^2 - 4\gamma(1-\gamma)\theta^2 - 4(1-\gamma)^2\theta^2}{\theta^6}$$

$$= \frac{(6\theta-4)(1-\gamma) + \gamma(2\theta-\gamma)}{\theta^4},$$

$$Skewness = \sqrt{(\beta_1)} = \frac{E(X^3)}{Var(X)^{3/2}} = \frac{6\theta^4(4(1-\gamma)\theta + \gamma\theta)}{(2\gamma\theta^3 + 6(1-\gamma)\theta^3 - \gamma^2\theta^2 - 4\gamma(1-\gamma)\theta^2 - 4(1-\gamma)^2\theta^2)^{3/2}}$$

$$Kurtosis = \beta_2 = \frac{E(X^4)}{(Var(X))^2}/(()) = \frac{(24\theta^6(5(1-\gamma)\theta + \theta\gamma)}{(2\gamma\theta^3 + 6(1-\gamma)\theta^3 - \gamma^2\theta^2 - 4\gamma(1-\gamma)\theta^2 - 4(1-\gamma)^2\theta^2)^2)},$$

$$C.V \ = \ K = \frac{\sqrt{Var(X)}}{E(X)} = \frac{\sqrt{2\gamma\theta^3 + 6(1-\gamma)\theta^2 - \gamma^2\theta^2 - 4\gamma(1-\gamma)\theta^2 - 4(1-\gamma)^2\theta^2}}{2(1-\gamma)\theta + \theta\gamma)}$$

$$= \frac{\sqrt{2\gamma\theta + 6(1-\gamma) - \gamma^2 - 4\gamma(1-\gamma) - 4(1-\gamma)^2}}{2-\gamma}$$

The moment generating function of the $TPFD$ is determined as follows

$$M(s) = \int e^{sx} f(x)dx = \frac{\theta^2\big((1-\gamma)\theta + \theta\gamma - s\gamma\big)}{\big((1-\gamma)\theta + \gamma\theta\big)(\theta-s)^2}, s < \theta$$

$$= \frac{\theta(\theta - s\gamma)}{(\theta-s)^2}$$

its characteristic function is obtained by replacing t with it in the last equation.

The ith incomplete moments of TPFD is determined as follows

$$T_i(s) = \int_0^s t^i f(x)dx = \frac{(1-\gamma)\theta\Gamma(2+i) + \theta\gamma\Gamma(1+i) - (1-\gamma)\theta\Gamma(2+i,s\theta\gamma) - \gamma\theta\Gamma(1+i,s\theta\gamma)}{\theta^i(1-\gamma)\theta + \gamma\theta^{i+1}}$$

where $\Gamma(\alpha,x) = \int_{-x}^{\infty} t^{\alpha-1} e^{-t} dt$. We have first incomplete moments $T_1(s)$ in above equation when $i = 1$ which used to calculate the mean residual life and the mean waiting time which is, respectively, defined as follows

$$\Psi(s) = \frac{1 - T_1(s)}{S(x;\theta,\gamma) - 1}$$

$$M_1(s) = \frac{1 - T_1(s)}{F(x;\theta,\gamma)}$$

Another uses of $T_1(s)$ is to calculate Bonferroni and Lorenz curves which are, respectively, defined as follows

$$L(p) = \frac{T_1(x)}{E(X)}$$

$$B(p) = \frac{T_1(x_p)}{pE(X)},$$

Where(x_p) is the quantile function of NTPFD.

Stochastic orders

Stochastic Order is an order of -largeness -on random variables. More broadly, stochastic orders are orders that are used to compare random variables, or probability distributions or measurements.

Now, we consider 2 random variables V and W . Then V is said smaller than W in the :

- Likelihood ratio order $(V <_{lr} W), if \frac{f_v(x)}{f_w(x)}$ is decreasing in x

-Hazard rate order $(V \leq_{hr} W), if h_v(x) \geq h_w(x), \forall x$

- Stochastic order $(V <_S W), if F_v(x) < F_w(x), \forall x$

- Convex order $(V \leq_{cx} W)$, if for all convex functions $\phi, E[\phi(V)] \leq E[\phi(W)]$(expectation exist).

Theorem 1. Let $V, W \sim TPFD$ be two random variables. If

$(1-\gamma_1)\theta_1\gamma_2 \leq (1-\gamma_2)\theta_2\gamma_1$ $, and \theta_1 \geq \theta_2$ then:$V <_{lr} W;\ V <_{hr} W;\ V <_s W and V \leq_{cx} W$

Proof. We have:

$$\frac{f_v(x)}{f_w(x)} = \frac{\frac{\theta_1{}^2(\gamma_1+(1-\gamma_1)\theta_1 t)exp(-\theta_1 t))}{\gamma_1+(1-\gamma_1)\theta_1 t}}{\frac{\theta_2{}^2(\gamma_2+(1-\gamma_2)\theta_2 t)exp(-\theta_2 t)}{\gamma_2+(1-\gamma_2)\theta_2 t}}$$

To keep it simple, we use $ln\frac{f_v(x)}{f_w(x)}$, which we find after derivation:

$$\frac{d}{dx} ln\frac{f_v(x)}{f_w(x)} = \frac{(1-\gamma_1)\theta_1\gamma_2 - (1-\gamma_2)\theta_2\gamma_1}{(\gamma^1 + (1-\gamma^1)\theta^1 t) + (\gamma_2 + (1-\gamma_2)\theta_2 t)} - (\theta_1 - \theta_2)$$

In this regard, if $\theta_1\gamma_2 \leq \theta_2\gamma_1 and \theta_1 \geq \theta_2$, we have$\frac{d}{dx}$ ln $(\frac{f_v(x)}{f_w(x)}) \leq 0$. It means that

$V <_{lr} W$. Moreover, we know that $V <_{lr} W \Rightarrow V <_{hr} W \Rightarrow V <_s W$ and $V \leq_{cx} W \Leftrightarrow V <_s W (if E[V] = E[W])$, which the Theorem 1is proved.

Entropies

There is general agreement that entropy and information can be used to calculate the degree of uncertainty in a probability distribution. However, many correlations have been generated from the characteristics of entropy. The entropy of a random variable X is a measurement of the variability of uncertainty. The entropy of Rényi is defined as:

$$I_R(s)=\frac{1}{(1-s)}log\int_0^{\infty} f^s\,(x)dx$$

Were s(integer)>0 et s≠1. For the NTPFD, we have:

$$I_R(s)=\frac{1}{(1-s)}\log\left(\int_0^{\infty}\left(\frac{\theta^2(\gamma+(1-\gamma)\theta x)\exp\,(-\theta x))}{\gamma\theta+(1-\gamma)\theta}\right)^s dx\right)$$

$$=\frac{1}{(1-s)}\log\left(\int_0^{\infty}\frac{\theta^{s\,2}}{(\gamma\theta+(1-\gamma)\theta)^s}(\gamma+(1-\gamma)\theta x)^s e^{-\theta xs}dx\right)$$

We observe that

$$\int_0^{\infty}\frac{\theta^{s\,2}}{(\gamma\theta+(1-\gamma)\theta)^s}(\gamma+(1-\gamma)\theta x)^s e^{-\theta xs}dx=\frac{\theta^{s\,2}}{(\gamma\theta+(1-\gamma)\theta)^s}\sum_{i=0}^{n}\frac{n!\,(\gamma)^i((1-\gamma)\theta)^{n-i}}{(n-i)!\,i!}\int_0^{\infty}x^{n-i}\,e^{-\theta xs}dx$$

where

$$\int_0^{\infty}x^{n-i}\,e^{-\theta xs}dx=\frac{-1}{s\theta}\Gamma(n+1-i,s\theta x)(s\theta)^{i-n}$$

Now, the Rényi entropy observes as

$$I_R(s)=\frac{1}{(1-s)}log\left(\frac{\theta^{s\,2}}{(\gamma\theta+(1-\gamma)\theta)^s}\sum_{i=0}^{n}\frac{n!\,(\gamma)^i((1-\gamma)\theta)^{n-i}}{(n-i)!\,i!}\frac{(s\theta)^{i-n}\Gamma(n-i+1)}{(s\theta)^{n-i}}\right).$$

Quantile Function

It may be noted that $F(x;\theta,\gamma)$ is continuos and strictly increasing, so we for the quantile function of T is defined:

$$Q_X(u)=x_u=F^{-1}(u;\theta,\gamma),\qquad u\in[0.1]$$

For $u=F(x;\theta,\gamma)$, we give an explicit expression for $Q_X(u)$ in terms of the Lambert W fuction in the following theorem and results.

Theorem 2. For any θ,γ>0 , the $Q_X(u)$of the NTPFD is

$$Q_X(u)\;=\;x_u=\frac{-\gamma\theta-(1-\gamma)\theta-W_{-1}((u-1)(\gamma\theta+(1-\gamma)\theta)e^{-(\gamma\theta+(1-\gamma)\theta)}}{(1-\gamma)\theta^2}\qquad,u\in[0.1]$$

$$=\frac{-\gamma-(1-\gamma)-W_{-1}((u-1)(\gamma+(1-\gamma))e^{-(\gamma\theta+(1-\gamma)\theta)}}{(1-\gamma)\theta}$$

$$=\frac{-1-W_{-1}((u-1)e^{-\theta}\,)}{(1-\gamma)\theta}$$

Where W_{-1} is the negative branch .

Proof. For any $\theta>0,0<\gamma<1\;let\;0<u<1$.We will solve the equation F_{TPFD}(t) with respect to x, by following the steps bellow:

$$e^{-\theta x}(\theta\gamma + (1-\gamma)\theta + (1-\gamma)\theta^2 x) = (\theta)(1-u). \qquad (*)$$

We multipling the both sides by $[-exp-(\theta)]$ of the equ(*) , we get :

$$-e^{-\theta t-(\gamma\theta+(1-\gamma)\theta)}(\theta\gamma + (1-\gamma)\theta + (1-\gamma)\theta^2 t) = (u-1)(\theta\gamma + (1-\gamma)\theta)e^{(-\theta)}$$

By using the definition of Lambert W function $(W(z)exp(W(z)) = z)$, we observe that $-(\theta + (1-\gamma)\theta^2 t)$ is the Lambert W function of the real argument$(u-1)(\theta)e^{-(\gamma\theta+(1-\gamma)\theta)}$. So, we have:

$$W((u-1)(\theta\gamma + (1-\gamma)\theta)e^{-(\gamma\theta+(1-\gamma)\theta)}) = -(\theta\gamma + (1-\gamma)\theta + (1-\gamma)\theta^2 t).$$

$$= (\theta + (1-\gamma)\theta^2 t)(**)$$

In addition, for any $\theta, \gamma, t > 0$ it's obviously that $\theta + (1-\gamma)\theta^2 t > 0$ and it also checked that $(u-1)(\theta\gamma + (1-\gamma)\theta)e^{(-\theta)}\varepsilon(-e^{-1}, 0)$ since 0< $u < 1$. Since , by taking into account the properties of the negative branch W_{-1} of the Lamber W function, son the equ above(**) become:

$$W_{-1}((u-1)(\theta\gamma + (1-\gamma)\theta)e^{(-\theta)}\ ) = -(\theta\gamma + (1-\gamma)\theta + (1-\gamma)\theta^2 t)$$

$$= -(\theta + (1-\gamma)\theta^2 t).$$

This in turn means the result that given before in Theorem 2 is complete.

4. Fuzzy reliability

Let X is a continuous random variable that represents a system's failure time (component). The fuzzy dependability can then be calculated using the fuzzy probability in formula (see Chen et al. (2001)).

$$R_F(t) = P(T > t) = \int_t^{\infty} \mu(x) f_{NTPFD}\,(x)dx, 0 \le t \le x < \infty,$$

where $\mu(x)$ is a membership function that describes the degree to which each element of a given universe belongs to a fuzzy set. Now, assume that $\mu(x)$ is

$$\mu(x) = \begin{cases} 0 & , \quad x \le t_1 \\ \dfrac{(x-t_1)}{(t_2-t_1)}, & 0 \le t_1 < x < t \\ 1 & , x \ge t_2 \end{cases}$$

For $\mu(x)$, by the computational analysis of the function of fuzzy numbers, the lifetime $x(\alpha)$ can be obtained corresponds to a certain value of $\alpha - Cut, \alpha \in [0,1]$, can by obtained as: $\mu(x) = \alpha \rightarrow \frac{x-t_1}{t_2-t_1} = \alpha$, then

$$\begin{cases} x(\alpha) \le t_1 & , \quad \alpha = 0 \\ x(\alpha) = t_1 + \gamma(t_2 - t_1) & ,0 < \alpha < 1 \\ x(\alpha) \ge t_2 , \alpha = 1 & \end{cases}$$

As a result, the fuzzy reliability values may be determined for all α values. The fuzzy dependability of the NTPFD is determined by the fuzzy reliability definition. The fuzzy reliability of the NTPFD can be define as,

$$R_F(t) = (1 + (1-\gamma)\theta t_1)e^{-\theta t_1} - (1 + (1-\gamma)\theta\alpha)e^{-\alpha\theta}$$

Then $R_F(t)_{\alpha=0} = 0$.

5. Applications: real data analysis

Two applications are now proposed to illustrate the usefulness of the proposed model. More precisely, we explore the tuning behavior of the NTPLD compared to two-parameter Lindley (Shanker and Ghebretsadik 2013) , Quasi Lindley (Benatmane et al.2021), Power XLindley (Meriem et al.2022), TPQED (Boussaba et al.2024) distributions. For this, we estimate the unknown parameters of the respective model using the maximum likelihood method and consider their corresponding standard errors (SE), the estimated log likelihoods (- 2logL), the values of AIC (Akaike information criterion)AICC(Akaike information criterion correction) ,and BIC (Bayesian information criterion).

Data Set 1: The data set 1 represents an uncensored data set corresponding to remission times (in months) of a random sample of 128 bladder cancer patients reported by Lee and Wang (2003)

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64,3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14,79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93,11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25,8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76,12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69.

Model	θ	γ	AIC	BIC	-2LL	AICC
two-parameter Lindley	0.1283	30.1556	833.328	839.0321	829.328	833.424
Quasi Lindley	0.31603	0.041835	1196.427	1202.131	1192.427	1196.523
Power XLindley	1.3886	0.261294	1127.513	1133.217	1123.513	1127.609
TPQED	0.18857	88.3213	841.4313	847.1353	837.4313	841.5273
NTPFD	0.10698	0.997623	**833.2282**	**838.9322**	**829.2282**	**833.3242**

Data set 2: This data set gives the population of the United States (in millions) as recorded by the decennial census for the period 1790--1970(McNeil (1977)). 3.93, 5.31, 7.24, 9.64, 12.90, 17.10, 23.20, 31.40, 39.80, 50.20, 62.90, 76.00, 92.00, 105.70, 122.80, 131.70, 151.30, 179.30, 203.20

Model	θ	γ	AIC	BIC	-2L	AICC
two-parameter Lindley	0.021668	44.2597	203.7026	205.5914	199.7026	204.4526
Quasi Lindley	0.04057	0.20435	336.5028	338.3917	332.5028	337.2528
New quasi Lindley	0.02478	0.0010298	204.5499	206.4388	200.5499	205.2999
Power XLindley	1.0557	0.19355	251.3029	253.1917	247.3029	252.0529
TPQED	0.04027	94.2316	214.4968	216.3856	210.4968	215.2468
NTPFD	0.01387	0.997565	**203.4288**	**205.3177**	**199.4288**	**204.1788**

6. Conclusion

In this paper we have shown how probability distributions can be constructed without adding additionalparametersor using the usual generalizations techniques. The proposed distribution is called the TPFD. It can be seen that the TPFD has many desirable properties. We have derived precise and explicit expressions for many characteristics, inparticular moments, reliability parameters and asymptotic distributions of order statistics. Inaddition, TPFD, two-parameter L1, Quasi Lindley, Power XLindley, and TPQEDdistributions were fitted to two real data sets; and the results showed that the TPFD distribution is a strong candidatewithtwoparameter distribution.

Declaration of Author Contributions

The authors declare that they have contributed equally to the article. All authors declare that they have seen/read and approved the final version of the article ready for publication.

Declaration of Conflicts of Interest

All authors declare that there is no conflict of interest related to this article.

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